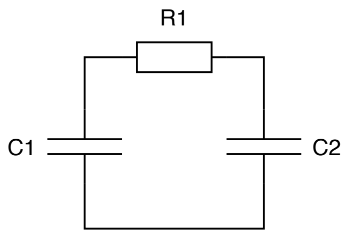
There is also the two capacitor paradox to consider. Take a 100V 100uF capacitor and attach it to another empty 100uF capacitor. The energy on the 100V capacitor is 0.5J. The two capacitors then equalize voltage so they have 50V each. The energy on both capacitors is now 0.125J each, giving a total of 0.25J. Where did the extra 0.25J go? If it's possible for the energy to simply disappear, then conversely it must be possible for energy to appear.

The energy on the charged cap before the experiment is

$$E_1 = \frac{1}{2}CU^2 = 0.5\,\mathrm{J}$$

What will the voltage on the two caps be after equalization?



Let's write down the differential equation describing the equalization process and thus see what happens.

$$U_1 = RI + U_2$$

in which current I is positive when going clockwise. We then have

$$\frac{Q_1}{C_1} = RI + \frac{Q_2}{C_2} \tag{1}$$

with

$$\frac{dQ_1}{dt} = -I \qquad \frac{dQ_2}{dt} = I$$

Let's diferentiate Eq. 1. This gives

$$\begin{aligned} \frac{1}{C_1}\dot{Q}_1 &= R\dot{I} + \frac{1}{C_2}\dot{Q}_2 \\ -\frac{1}{C_1}I &= R\dot{I} + \frac{1}{C_2}I \\ 0 &= R\dot{I} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right) I \end{aligned}$$

$$\dot{I} + \frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) I = 0 \tag{2}$$

That's a homogeneous first order differential equation. Let's solve this with the following approach (educated guess).

$$\begin{aligned} I\left(t\right) &= I_0 e^{-kt} \\ \dot{I}\left(t\right) &= -I_0 k e^{-kt} \end{aligned}$$

Substituting both into Eq. 2 gives

$$\begin{aligned} -I_0 k e^{-kt} + \frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) I_0 e^{-kt} &= 0 \\ -k e^{-kt} + \frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) e^{-kt} &= 0 \\ \left(\frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) - k\right) e^{-kt} &= 0 \end{aligned}$$

This equation is true for all t only if

$$k = \frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

This gives us the current for the equalization.

$$\boxed{I\left(t\right) = I_0 e^{-\left(\frac{1}{R}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right)t}} \qquad \text{with} \qquad I_0 = \frac{U_0}{R}$$

in which U_0 is the start voltage on the left cap (in your example 100V) and R is the resistance of the wire connecting the two caps (there always be some although small).

What about the charges on the two caps.

$$
\begin{aligned}
Q_1(t) &= -\int I \, dt \\
Q_1(t) &= -\int I_0 e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t} \, dt \\
Q_1(t) &= -\left[\frac{1}{-\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} I_0 e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t}\right]_0^t + C \\
Q_1(t) &= -\left(\frac{1}{-\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} I_0 e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t} - \frac{1}{-\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} I_0\right) + C \\
Q_1(t) &= \frac{1}{-\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} I_0 - \frac{1}{-\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} I_0 e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t} + C \\
Q_1(t) &= \frac{1}{-\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} I_0 \left(1 - e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t}\right) + C \\
Q_1(t) &= \frac{1}{-\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} \frac{U_0}{R} \left(1 - e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t}\right) + C \\
Q_1(t) &= -\frac{U_0}{\frac{1}{C_1}+\frac{1}{C_2}} \left(1 - e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t}\right) + C \\
Q_1(t) &= \frac{U_0}{\frac{1}{C_1}+\frac{1}{C_2}} \left(e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t} - 1\right) + C
\end{aligned}
$$

At $t = 0$ we have $Q_1 = C_1 U_0$ in which $U_0 = 100\,\mathrm{V}$. This gives

$$
\begin{aligned}
C_1 U_0 &= \frac{U_0}{\frac{1}{C_1}+\frac{1}{C_2}} (1-1) + C \\
C_1 U_0 &= C
\end{aligned}
$$

We finally get

$$
Q_1(t) = \frac{U_0}{\frac{1}{C_1}+\frac{1}{C_2}} \left(e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t} - 1\right) + C_1 U_0
$$

and by dividing by C_1.

$$
U_1(t) = \frac{1}{C_1} \frac{U_0}{\frac{1}{C_1}+\frac{1}{C_2}} \left(e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t} - 1\right) + U_0
$$

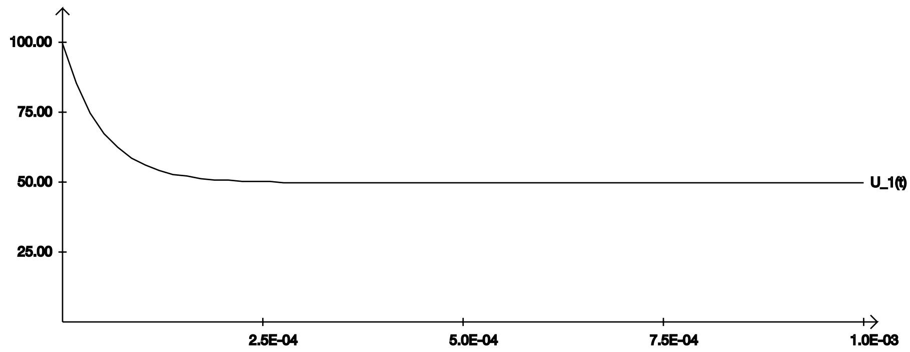


Figure 1: C_1=100E-6; C_2=100E-6; R=1; U_0=100

This shows that the voltage in fact will be 50V at the end. So we indeed have only 0.25J left in the caps!? Where did the remaining 0.25J go to? Let's see.

$$U_R = RI$$

$$\begin{aligned}
P_{loss} &= RI^2 \\
E_{loss} &= \int P_{loss}\, dt \\
E_{loss} &= \int R\left(I_0 e^{-\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t}\right)^2 dt \\
E_{loss} &= \int R\frac{{U_0}^2}{R^2} e^{-2\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t}\, dt \\
E_{loss} &= \frac{{U_0}^2}{R}\int e^{-2\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t}\, dt \\
E_{loss} &= \frac{{U_0}^2}{R}\left[\frac{1}{-2\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)} e^{-2\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)t}\right]_0^\infty \\
E_{loss} &= \frac{{U_0}^2}{R}\left(-\frac{1}{-2\left(\frac{1}{R}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right)}\right) \\
E_{loss} &= \frac{1}{2}{U_0}^2\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}
\end{aligned}$$

For $C = C_1 = C_2$ we get

$$E_{loss} = \frac{1}{4}C{U_0}^2 = 0.25\,\mathrm{J}$$

There you have your loss. No paradox at all. :-) If you want to make energy flow into your system you have to come up with negative resistance.